Some Extensions to the Analysis of the Internalization of Nonhomogeneous Design Effects in Recycling Systems

Kazuhiko NISHIMURA*

Abstract

In this note, we show that efficiency as well as the dynamical property of the equilibrium of a recycling system, with net price minimization of individual producers and normative pricing of nonhomogeneous wastes by the monopolistic recycler, will not be affected even if we considered higher level (higher than the second) moments averseness in the recycling activity.

Key words: Recycle Desigu, General Equilibrium, Efficiency, Stability

1 Introduction

Nishimura (2007) investigated an autonomous recycling system from both the static and dynamical aspects using a simple three sector general equilibrium model, which consists of many identical primary producers, a representative consumer, and a recycler. A recycler is a monopoly although it is assumed to be subject the marginal cost regulation, and it utilizes the consumed product and recycles secondary materials back to the primary producers. A producer produces consumer goods with a recycle design that minimizes the net price to the product. A consumer's utility depends on the amount of consumption but not on the recycle design of the product.

The preceding paper first considers the model in which the producers modify the design in a uniform manner as if there were a single agent. This assumption, although very restrictive, ensures the homogeneity of recycle design at every stage. Then, it is relaxed to consider the existence of different designs at the same time. A vanguard producer who attempts to exercise control over the adjustment of the design of his/her products, according to his incentive for lowering his product's net price, is then introduced. The rest of the producers follow this design as soon as they realize that this design is advantageous with respect to its net price. The monopolistic recycler will purchase the consumed product (waste) of any design

*Associate Professor, Faculty of Economics, Nihon Fukushi University

on the basis of its marginal revenue.

The analysis for the design diversity effect in the preceding paper is restricted to the case when the design-in-effect to the recycler is represented by the mean and the variance of wastes with respect to multiple (two actually) designs. This note therefore extends the analysis of the entire diversity effects with higher and normalized central moments, and show that the mean and the variance are the only two essential effects after all. In the remaining sections we first introduce briefly the basic models of this recycling system with multiple designs, and then show analytically that all of the higher moments besides mean and variance vanish in the diversity effects with respect to efficiency as well as to dynamical stability.

2 The Model

Here, we briefly describe the essence of the base model as it is explained in detail in the preceding paper. First we put here the basic assumptions for the autonomous recycling system with multiple (nonhomogeneous) designs.

Assumption 1 Nonhomogenous Design

1. *There exists a vanguard producer i who makes changes to the design if the net price of the consumer goods produced by i is lowered by doing so.*
2. *The recycler reveals the marginal revenue from wastes (as the normative price of wastes) from the consumer goods produced by i.*
3. *The other producers simultaneously follow i's design immediately after the observation of the advantageousness of i's design.*
4. *Moments in recycle design affect the recycling activity.*

The primary producer's production function is described as below, where, l and y denote the physical amount of labor and secondary input, respectively. As it is shown, the producer jointly produces x amount of consumer goods and σ amount of design via these inputs.

$$x = X(y,l)$$
$$\sigma = Z(y,l)$$

Note that X is homogeneous of the first order so that it is consistent with the implicit assumption of many producers with perfect competition. We also assume that Z is homogeneous of null degree. Thus, if y and l represent the amount for the industry, then σ should also represent the amount of the whole industry i.e., the design is implicitly homogeneous. Here instead, we assume that inputs are individual and that σ is the vanguard's design.

We let k denote the amount of labor used in the recycling activity and write the recycler's production function which is affected by the waste's design-in-effect s as follows.

$$y = \begin{cases} Y(x,k;s) & k \geq \bar{k} \\ 0 & \text{otherwize} \end{cases}$$

Since this recycler is a monopolist with a large fixed cost the recycler will not operate unless k exceeds certain threshold $\bar{k}$. Assume Y is homogeneous of the first order with respect to x and $k-\bar{k}$.

Since design is homogeneous at the equilibrium, we let ρ, δ and $\omega(s)$ denote the prices of goods, labor and wastes with design σ, respectively. By Assumption 1.1 and the Euler's rule, we have the following identity.

$$\rho Y_x(x,k;s) = \omega(s) \quad (1)$$
$$\rho Y_k(x,k;s) = \delta \quad (2)$$
$$Y(x,k;s) = Y_x(x,k;s)x + Y_k(x,k;s)\{k-\bar{k}\} \quad (3)$$

Thus, we arrive at the following identity at the equilibrium.

$$\rho Y_s(x,k;s) = \omega_s(s)x \quad (4)$$

On the basis of the Assumption 1.4, we may write the design-in-effect s within the recycling activity using the vanguard's amount of production ξ, the rest of producers' amount of production $\bar{x}$, and the rest of producers' design $\bar{s}$. We note this as below to include all possible moments that may affect the recycling activity.

$$s = S\left(m, \{\mu_i\}_{i=2}^{\infty}, \left\{\left\{\bar{\mu}_{i(j)}\right\}_{i=2}^{\infty}\right\}_{j=2}^{\infty}\right) \quad (5)$$

Here we use the following definitions.

$$p = \frac{\xi}{\xi+\bar{x}} = \frac{\xi}{x} = \frac{1}{n} \quad (6)$$
$$m = p\sigma + \{1-p\}\bar{s} \quad (7)$$
$$\mu_i = p\{\sigma-m\}^i + \{1-p\}\{\bar{s}-m\}^i \quad (8)$$
$$\mu_{i(j)} = \mu_i/\mu_j^{i/j} \quad (9)$$

The production share of the vanguard with n identical producers is denoted by p, and the mean of design in the wastes is denoted by m. The i th moment is denoted by μ_i, and the i th moment standardized by the j th moment is denoted by $\mu_{i(j)}$. Note that $\mu_{3(2)}$ will be the skewness and $\mu_{4(2)}$ will be the kurtosis. Also, we introduce the normalized standardized moments for cases such as $\lim_{\sigma\to\bar{s}}\mu_{i(j)} \neq 0$, as follows:

$$\bar{\mu}_{i(j)} = \mu_{i(j)} - \lim_{\sigma\to\bar{s}}\mu_{i(j)}$$

According to the analysis given in Nishimura (2007), it is possible to show that, if $\sigma = s$ (that is, the design is always homogeneous), then the first order conditions obtained through the market mechanism, including Eqns. (2-3) and (4), are identical to those of the social optimum, in which case the efficiency is restored. In the following section we show that these first order conditions will not be affected even though we consider Eqn. (5) in the neighborhood of the equilibrium.

3 Efficiency

Let the price of wastes of vanguard's design, while that of the others is $\bar{s}$, be denoted by $\omega^i(\sigma;\bar{s})$. Then according to the Assumption 1.2, this equals to the marginal revenue of the recycler i.e.,

$$w^i(\sigma;\bar{s}) = \rho\frac{\partial Y(x,k,s)}{\partial \xi} = \rho\frac{\partial Y}{\partial x}\frac{\partial x}{\partial \xi}+\rho\frac{\partial Y}{\partial s}\frac{\partial S}{\partial \xi} \quad (10)$$

Naturally, it is $\frac{\partial x}{\partial \xi} = 1$ because $x = \bar{x}+\xi$. Thus, we investigate how $\frac{\partial S}{\partial \xi}$ is affected at the equilibrium state. By Eqn.(moments), we have

$$\frac{\partial S}{\partial \xi} = S_m\frac{\partial m}{\partial \xi}+\sum_{i=2}^{\infty} S_{\mu i}\frac{\partial \mu_i}{\partial \xi}+\sum_{j=2}^{\infty}\sum_{i=2}^{\infty} S_{\bar{\mu}_{i(j)}}\frac{\partial \bar{\mu}_{i(j)}}{\partial \xi}$$

where, $S_m = \frac{\partial S}{\partial m}$, $S_{\mu_i} = \frac{\partial S}{\partial \mu_i}$, $S_{\bar{\mu}_{i(j)}} = \frac{\partial S}{\partial \bar{\mu}_{i(j)}}$. We let these quotients to be nonzero constant to avoid inessential complexity.

By definitions (7-9), we have the following identities.

$$\lim_{\sigma\to\bar{s}} m = \bar{s}$$

$$\lim_{\sigma\to\bar{s}} \mu_i = 0$$

$$\lim_{\sigma\to\bar{s}} \bar{\mu}_{i(j)} = 0$$

Because σ and ζ are independent variables, we arrive at the following result.

$$\lim_{\sigma\to\bar{s}}\frac{\partial S}{\partial \xi} = S_m\frac{\partial \bar{s}}{\partial \xi} = 0 \quad (11)$$

Thus, we have, by Eqn. (10),

$$\lim_{\sigma\to\bar{s}} \omega^i(\sigma;\bar{s}) = \rho Y_x(x,k,s) = \omega(s)$$

where we see that the static first order efficiency conditions are maintained.

4 Stability

We focus on the *ceteris paribus* dynamics, that is, the following recursive process of the design search,

$$\sigma' = \sigma - f_\sigma(\sigma;\bar{s})d$$

is executed so as to diminish the net price of goods $f(\sigma;\bar{s}) = \phi(\sigma) - \omega^i(\sigma;\bar{s})$ where all other prices are kept unchanged until the process finds its extremum. Note that $-f_\sigma(\sigma;\bar{s})$ is the direction of descent and d is some small number. At the equilibrium, $f(\sigma;\bar{s})$ must be concave with respect to σ else the equilibrium will be unstable.

Thus, we examine the sign of $f_{\sigma\sigma}(\sigma;\bar{s})$ at the equilibrium. By Eqn. (4) we have

$$\rho Y_{sx}(x,k;s) = \omega_s(s)$$
$$\rho Y_{ss}(x,k;s) = x\omega_{ss}(s)$$

By Eqns. (10) and (11), we have what follows below.

$$\lim_{\sigma\to\bar{s}} w^i_{\sigma\sigma} = \omega_{ss} \lim_{\sigma\to\bar{s}} \frac{\partial S}{\partial\sigma}\left\{\frac{\partial S}{\partial\sigma} + 2x\frac{\partial^2 S}{\partial\xi\partial\sigma}\right\} + \omega_s \lim_{\sigma\to\bar{s}}\left\{\frac{\partial^2 S}{\partial\sigma^2} + x\frac{\partial^3 S}{\partial^2\sigma\partial\xi}\right\} \tag{12}$$

We will then examine every term appeared in the rhs of Eqn. (12).

By definition (5) we naturally have

$$\frac{\partial S}{\partial\sigma} = S_m\frac{\partial m}{\partial\sigma} + \sum_{i=2}^{\infty} S_{\mu_i}\frac{\partial\mu_i}{\partial\sigma} + \sum_{j=2}^{\infty}\sum_{i=2}^{\infty} S_{\bar{\mu}_{i(j)}}\frac{\partial\bar{\mu}_{i(j)}}{\partial\sigma}$$

where,

$$\frac{\partial m}{\partial\sigma} = p = \frac{1}{n}$$
$$\frac{\partial\mu_i}{\partial\sigma} = i\{\bar{s}-\sigma\}^{i-1}\left\{p^{i+1} - p\{p-1\}^i - p^i\right\}$$
$$\frac{\partial\bar{\mu}_{i(j)}}{\partial\sigma} = \frac{ij-ji}{j\bar{s}-j\sigma}\,\frac{p\{p-1\}^i - \{p-1\}p^i}{\left\{p\{p-1\}^j - \{p-1\}^i p^j\right\}^{i/j}} = 0$$

Thus, we have

$$\lim_{\sigma\to\bar{s}}\frac{\partial S}{\partial\sigma} = S_m p = S_m\frac{1}{n} \tag{13}$$

Also, by definition (5) we naturally have

$$\frac{\partial^2 S}{\partial\sigma^2} = S_m\frac{\partial^2 m}{\partial\sigma^2} + \sum_{i=2}^{\infty} S_{\mu_i}\frac{\partial^2\mu_i}{\partial\sigma^2} + \sum_{j=2}^{\infty}\sum_{i=2}^{\infty} S_{\bar{\mu}_{i(j)}}\frac{\partial^2\bar{\mu}_{i(j)}}{\partial\sigma^2}$$

where,

$$\frac{\partial^2 m}{\partial\sigma^2} = 0$$
$$\frac{\partial^2\mu_i}{\partial\sigma^2} = -i\{i-1\}\{\bar{s}-\sigma\}^{i-2}\left\{p^{i+1} - p\{p-1\}^i - p^i\right\}$$
$$\frac{\partial^2\bar{\mu}_{i(j)}}{\partial\sigma^2} = 0$$

Hence,

$$\lim_{\sigma \to \bar{s}} \frac{\partial^2 S}{\partial \sigma^2} = -2S_{\mu_2} p\{p-1\} = 2S_{\mu_2}\left\{1-\frac{1}{n}\right\}\frac{1}{n} \tag{14}$$

Once again, by definition (5) we have

$$\frac{\partial^2 S}{\partial \sigma \partial \xi} = S_m \frac{\partial^2 m}{\partial \sigma \partial \xi} + \sum_{i=2}^{\infty} S_{\mu i} \frac{\partial^2 \mu_i}{\partial \sigma \partial \xi} + \sum_{j=2}^{\infty}\sum_{i=2}^{\infty} S_{\bar{\mu}_{i(j)}} \frac{\partial^2 \bar{\mu}_{i(j)}}{\partial \sigma \partial \xi}$$

where,

$$\frac{\partial^2 S}{\partial \sigma \partial \xi} = \frac{\partial p}{\partial \xi} = \frac{1}{x}\left\{1-\frac{1}{n}\right\}$$

$$\frac{\partial^2 \mu_i}{\partial \sigma \partial \xi} = i\{\bar{s}-\sigma\}^{i-1}\left\{p^{i-1}\{p+pi-i\}-\{p-1\}^{i-1}\{p+pi-1\}\right\}\frac{1}{x}\left\{1-\frac{1}{n}\right\}$$

$$\frac{\partial^2 \bar{\mu}_{i(j)}}{\partial \sigma \partial \xi} = 0$$

Hence,

$$\lim_{\sigma \to \bar{s}} \frac{\partial^2 S}{\partial \sigma \partial \xi} = S_m \frac{1}{x}\left\{1-\frac{1}{n}\right\} \tag{15}$$

Finally, by definition (5) we have

$$\frac{\partial^3 S}{\partial \sigma^2 \partial \xi} = S_m \frac{\partial^3 m}{\partial \sigma^2 \partial \xi} + \sum_{i=2}^{\infty} S_{\mu_i} \frac{\partial^3 \mu_i}{\partial \sigma^2 \partial \xi} + \sum_{j=2}^{\infty}\sum_{i=2}^{\infty} S_{\bar{\mu}_{i(j)}} \frac{\partial^3 \bar{\mu}_{i(j)}}{\partial \sigma^2 \partial \xi}$$

where,

$$\frac{\partial^3 m}{\partial \sigma^2 \partial \xi} = 0$$

$$\frac{\partial^3 \mu_i}{\partial \sigma^2 \partial \xi} = -i\{i-1\}\{\bar{s}-\sigma\}^{i-2}\left\{p^{i-1}\{p+pi-i\}-\{p-1\}^{i-1}\{p+pi-1\}\right\}\frac{1}{x}\left\{1-\frac{1}{n}\right\}$$

$$\frac{\partial^3 \bar{\mu}_{i(j)}}{\partial \sigma^2 \partial \xi} = 0$$

Thus,

$$\lim_{\sigma \to \bar{s}} \frac{\partial^3 S}{\partial \sigma^2 \partial \xi} = 2S_{\mu_2}\frac{1}{x}\left\{1-\frac{1}{n}\right\}\left\{1-\frac{2}{n}\right\} \tag{16}$$

Now, by using Eqns. (13), (14), (15), and (16) to Eqn. (12), we arrive at the following terms.

$$\lim_{\sigma \to \bar{s}} \omega^i_{\sigma\sigma} = S_m \omega_{ss} - \left\{1-\frac{1}{n}\right\}^2 \{-2S_{\mu_2}\omega_s + S_m \omega_{ss}\}$$

Because it is $\lim_{\sigma \to \bar{s}} \phi_{\sigma\sigma} = \phi_{ss}$ by the coidentity of the producers, we obtain the terms below.

$$\lim_{\sigma\to\bar{s}} f_{\sigma\sigma} = \phi_{ss} - S_m\omega_{ss} + \left\{1 - \frac{1}{n}\right\}^2 \{-2S_{\mu_2}\omega_s + S_m\omega_{ss}\}$$

As we normalize the mean effect i.e., $S_m = 1$, and write $S_{\mu_2} = -\eta$, we have the same terms as in Nishimura (2007).

$$\lim_{\sigma\to\bar{s}} f_{\sigma\sigma} = \phi_{ss} - \omega_{ss} + \left\{1 - \frac{1}{n}\right\}^2 \{2\eta\omega_s + \omega_{ss}\}$$

5 Concluding Remarks

We have investigated how the efficiency and the dynamic property of equilibrium can be altered if we introduce diversity in recycle design and a recycler who is affected by the entire moments in design. The first order condition will not be influenced by any moment higher than the first (mean) since changes in design and changes in quantity are marginally independent. Therefore, we found that higher moments do not influence economic efficiency. We also found that dynamical (i.e., second order) condition will be influenced by any moments lower than the second (variance) since all terms regarding higher moments vanish as the vanguard design approaches the equilibrium. Thusly, we confirm that the findings obtained in the preceding paper were not only necessary but sufficient even if we extend to consider a recycler with averseness to arbitrary class of moments.

References

Nishimura, K. (2007), The Role and Internalization of Homogeneous and Nonhomogeneous Design Effects in Recycling Systems, *Metroeconomica,* Forthcoming.